Question 1

1 Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 5, 7, 9\}$, and $B = \{1, 2, 5, 6\}$. What is $A \cap B'$?

(a) $\{1, 2, 3, 5, 6, 7, 9\}$ (b) $\{1, 5, \}$ (c) $\{1, 2, 4, 5, 6, 8, 10\}$

(d) $\{3, 7, 9\}$ (e) $\{1, 3, 4, 5, 7, 9, 10\}$

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- *We have* $A = \{1, 3, 5, 7, 9\}$ *and* $B' = \{3, 4, 7, 8, 9, 10\}$

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- *We have* $A = \{1, 3, 5, 7, 9\}$ *and* $B' = \{3, 4, 7, 8, 9, 10\}$
- *The matching elements are shown in red* $A = \{1, 3, 5, 7, 9\}$ *and* $B' = \{3, 4, 7, 8, 9, 10\}$

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- *Hence* $\boxed{A \cap B' = \{3, 7, 9\}}$.

Question 2

2 Which of the following sets is equal to

$$\emptyset \cup (S \cup T)'$$

Here $\emptyset$ denotes the empty set.

(a) $\emptyset \cup S' \cup T'$ (b) $\emptyset$ (c) $S \cup T$ (d) $\emptyset \cap S' \cap T'$ (e) $S' \cap T'$

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- *Therefore* $\emptyset \cup (S \cup T)' = \emptyset \cup (S' \cap T') = S' \cap T'$.

Question 3

3 A group of 30 students who exercise regularly, were asked about their exercise preferences. Fifteen students said they swam, 20 students said they ran, and 5 students said they neither swam nor ran. How many students said they did both types of exercise?

(a) 5 (b) 10 (c) 15 (d) 20 (e) 25

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- *We have* $n(U) = 30$. *Let* R *be the set of all students in the study who run and let* S *be the set of all students in the study who swim. The information given can be summarized as follows:*

$$n(U) = 30, \quad n(R) = 20, \quad n(S) = 15, \quad n((S \cup R)') = 5.$$

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- *The inclusion-exclusion principle says that $n(S \cup R) = n(S) + n(R) - n(S \cap R)$. Filling in what we know, we get*

$$n(S \cup R) = 15 + 20 - n(S \cap R).$$

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- *The complement rule gives* $n(S \cup R) = n(U) - n((S \cup R)') = 30 - 5 = 25$*.*

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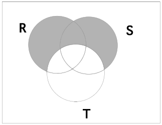
- *The complement rule gives* $n(S \cup R) = n(U) - n((S \cup R)') = 30 - 5 = 25$.
- *Substituting this in the above equation, we get*

$$25 = 15 + 20 - n(S \cap R) \quad \textit{or} \quad \boxed{n(S \cap R) = 10.}$$

Question 4

4



Which of the following sets is represented by the shaded region in the Venn Diagram above?

(a) $R \cap S \cap T'$ (b) $(R \cup S) \cap T'$ (c) $(R \cap S) \cup T'$
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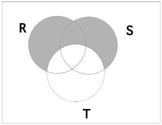

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- *If you shade all of* $R \cup S$ *and erase that part which intersects* T*, you will get the shaded region above. Therefore, the above shaded region is that of* $(R \cup S) \cap T'$*. If you prefer, you can draw the shaded regions corresponding to each answer using the techniques developed in class, and compare with that given one.*

Question 5

5 100 students were asked whether they like rock, folk, or classical music. Of these 55 liked rock, 45 liked folk, and 20 liked classical music. In addition 21 liked both rock and folk, 5 liked both rock and classical, and 10 liked both folk and classical music. If 18 people liked rock and folk but not classical music, how many liked none of these kinds of music?

(a) 10 (b) 11 (c) 12 (d) 13 (e) 14

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(a) 10 (b) 11 (c) 12 (d) 13 (e) 14

- *We can organize the information given in a table:*

U	100
R	55
F	45
C	20
$R \cap F$	21
$R \cap C$	5
$F \cap C$	10
$R \cap F \cap C'$	18

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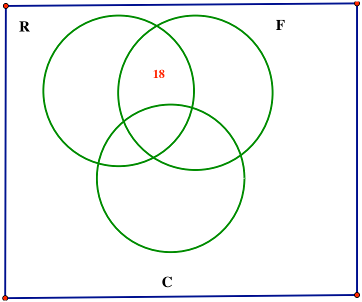

- *Starting with $R \cap F \cap C'$, we enter the numbers in a Venn diagram*

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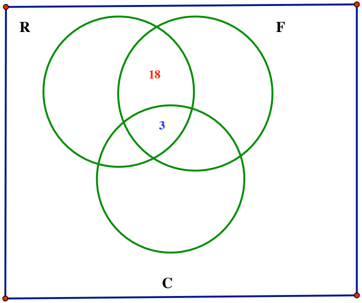

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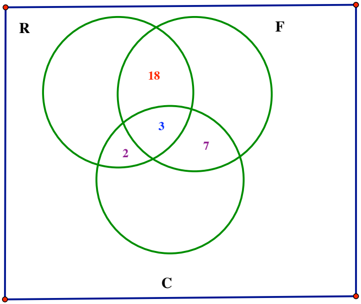

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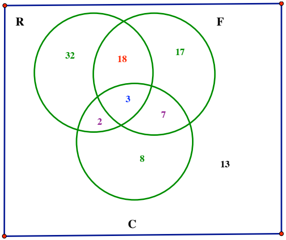

- *When we add the numbers in $R \cup F \cup C$ and subtract from 100 we get that the number in $(R \cup f \cup C)' = 13$.*

Question 6

6 How many 5 letter words can be made from the letters

WELOVEMATH

if letters cannot be repeated?

(a) $9!$ (b) 9^5 (c) 5^9 (d) $5!$ (e) $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$

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(a) 9! (b) 9^5 (c) 5^9 (d) 5! (e) $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$

- *We first discard the repeated letters to get 9 letters: WELOVMATH.*

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- *We first discard the repeated letters to get 9 letters: WELOVMATH.*
- *We have to form five letter words from these letters, in which letters are not repeated. This means that when forming such a word, we have 9 choices for the first letter, 8 choices for the second letter, seven choices for the third letter, etc....*

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- *We have to form five letter words from these letters, in which letters are not repeated. This means that when forming such a word, we have 9 choices for the first letter, 8 choices for the second letter, seven choices for the third letter, etc....*
- *Using the multiplication principle, the number of such words that we can form is:*

$$\underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5}.$$

Question 7

7 What is the numerical value of $P(6, 4)$?

(a) 30 (b) 15 (c) 360 (d) 10 (e) 120

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7 What is the numerical value of $P(6,4)$?

(a) 30 (b) 15 (c) 360 (d) 10 (e) 120

- $$P(6,4) = 6 \cdot 5 \cdot 4 \cdot 3 = 360.$$

Question 8

8 A stamp collector has 33 especially rare stamps (all different). She wants to give 11 of them to each of her three children, Susan Frank and Bob. In how many ways can she do this?

(a) $P(33,3)P(33,11)^3$ (b) $C(33,11)C(22,11)C(11,11)$

(c) $C(33,11)C(11,3)$ (d) $P(33,3)P(22,11)P(11,11)$ (e) $\frac{1}{3!}\frac{33!}{(11!)^3}$

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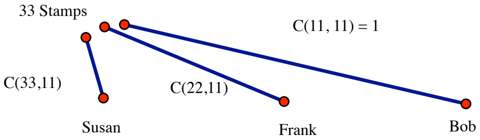
- *This is a reverse urn problem.*



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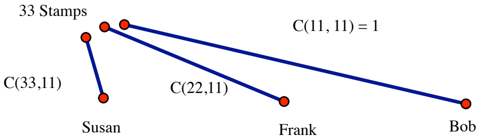

- *There are* $C(33,11)$ *ways to give Susan 11 of the 33 stamps.*

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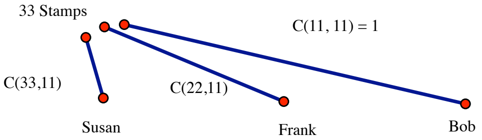

- *There are* $C(33,11)$ *ways to give Susan 11 of the 33 stamps.*
- *There are* $C(22,11)$ *ways to give Frank 11 of the remaining 22 stamps.*

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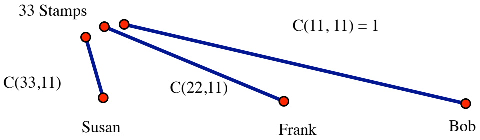

- *There are* $C(33,11)$ *ways to give Susan 11 of the 33 stamps.*
- *There are* $C(22,11)$ *ways to give Frank 11 of the remaining 22 stamps.*
- *There are* $C(11,11) = 1$ *way to give Bob 11 of the remaining 11 stamps.*

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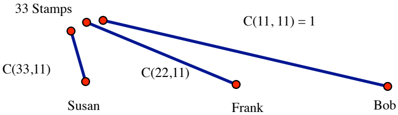
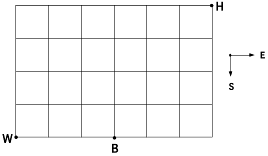


- *There are* $C(33,11)$ *ways to give Susan 11 of the 33 stamps.*
- *There are* $C(22,11)$ *ways to give Frank 11 of the remaining 22 stamps.*
- *There are* $C(11,11) = 1$ *way to give Bob 11 of the remaining 11 stamps.*
- *Using the multiplication principle, we see that there are* $C(33,11)C(22,11)C(11,11)$ *ways to complete the task.*

Question 9

9 The following is a part of a street map of Walkertown:



A woman visiting the city on business stays in a Hotel at H and will visit a business at W. How many different routes can she choose in walking to the business at W from her Hotel at H, if she wants to stop first at the bank at B? (Assuming she only wants to walk south or west.)

(a) $3!4!$ (b) $2^3C(7,4)$ (c) $C(7,3)$ (d) 12 (e) $P(7,3)$

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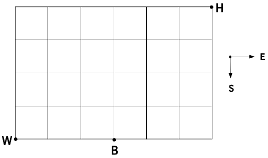

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- *(Number of routes from H to W) =(Number of routes from H to B) $\times$ (Number of routes from B to W).*

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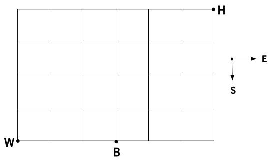

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- *(Number of routes from H to W)* =*(Number of routes from H to B)* $\times$ *(Number of routes from B to W).*
- *(Number of routes from H to W)* $= C(7,3) \times C(3,0) = C(7,3)$.

Question 10

10 How many different poker hands of 5 cards from a standard 52 card deck contains 3 cards of one denomination and 2 cards of a different denomination?

(a) $12 \cdot C(52,3) \cdot C(48,2)$ (b) $13 \cdot 12 \cdot C(4,3) \cdot C(4,2)$

(c) $13 \cdot 12 \cdot P(4,3) \cdot P(4,2)$ (d) $C(13,2) \cdot C(4,3) \cdot C(4,2)$ (e) $6 \cdot C(52,5)$

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(c) $13{\cdot}12{\cdot}P(4,3){\cdot}P(4,2)$ (d) $C(13,2){\cdot}C(4,3){\cdot}C(4,2)$ (e) $6{\cdot}C(52,5)$

- *The task of selecting such a hand can be broken into 4 steps:*

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(c) $13 \cdot 12 \cdot P(4,3) \cdot P(4,2)$ (d) $C(13,2) \cdot C(4,3) \cdot C(4,2)$ (e) $6 \cdot C(52,5)$

- *The task of selecting such a hand can be broken into 4 steps:*
- *Choose a denomination:* $C(13,1) = 13$ *ways,*

Question 10

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- *Choose a denomination:* $C(13,1) = 13$ *ways,*
- *Choose 3 cards from that denomination:* $C(4,3)$ *ways,*
- *Choose a different denomination:* $C(12,1)$ *ways,*
- *Choose 2 cards from that denomination:* $C(4,2)$ *ways.*
- *Multiplying the number of ways of completing each step gives: Number of hands with 3 cards from one denomination and 2 from another* $= 13 \cdot C(4,3) \cdot C(4,2) \cdot 12.$

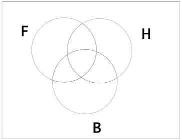
11(10 Pts) Let **FB** = {all students at Notre Dame who regularly attend football games}
Let **M** = {all male students at Notre Dame}
(a) Describe, in words, the set $FB \cap M'$.

(b) Describe, in words, the set $(FB \cap M')'$.

(c) A survey about game attendance was filled out by 10 students. The answers to each question, given by each student is shown below ("y" denotes "yes" and "n" denotes "no" and the first letter of each students name is given above their answers) :

	J	*M*	*P*	*S*	*A*	*C*	*B*	*Z*	*W*	*R*
Do you regularly attend football games	*y*	*y*	*n*	*y*	*y*	*y*	*y*	*y*	*n*	*y*
Do you regularly attend basketball games	*n*	*y*	*y*	*n*	*n*	*n*	*y*	*y*	*y*	*y*
Do you regularly attend Hockey games	*n*	*n*	*y*	*n*	*y*	*y*	*n*	*y*	*y*	*y*

Fill in the number of people surveyed who belong in each section of the following Venn diagram. Here, F denotes those who regularly attend football games, B denotes those who regularly attend basketball games and H denotes those who regularly attend hockey games.



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(a) Describe, in words, the set $FB \cap M'$.

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	J	M	P	S	A	C	B	Z	W	R
Do you regularly attend football games	y	y	n	y	y	y	y	y	n	y
Do you regularly attend basketball games	n	y	y	n	n	n	y	y	y	y
Do you regularly attend Hockey games	n	n	y	n	y	y	n	y	y	y

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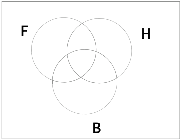

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Do you regularly attend basketball games	*n*	*y*	*y*	*n*	*n*	*n*	*y*	*y*	*y*	*y*
Do you regularly attend Hockey games	*n*	*n*	*y*	*n*	*y*	*y*	*n*	*y*	*y*	*y*

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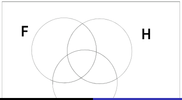

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- *We can also use DeMorgan's laws to see that*

$$(FB \cap M')' = FB' \cup M.$$

This set is obviously the set of students who either do not regularly attend football games or are male.

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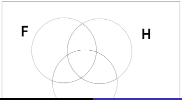

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Do you regularly attend Hockey games	*n*	*n*	*y*	*n*	*y*	*y*	*n*	*y*	*y*	*y*

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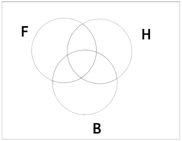

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Do you regularly attend football games	*y*	*y*	*n*	*y*	*y*	*y*	*y*	*y*	*n*	*y*
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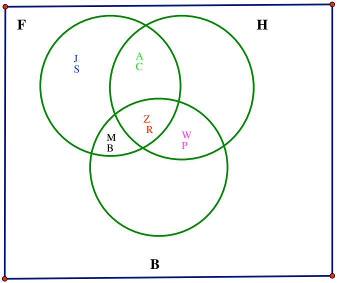

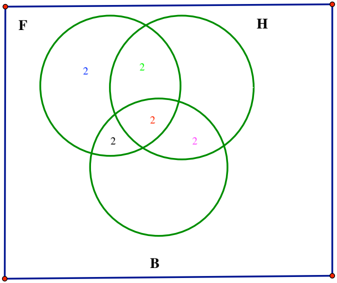

12(10 Pts) A factory ships ipods in boxes of 100. A quality control inspector chooses a sample of 5 ipods from the box for inspection prior to shipping. If no defectives are found, the box will be shipped. If at least one defective is found in the sample, the box will not be shipped.

(a) Suppose a box of 100 ipods contains exactly 20 defective ipods. In how many ways can a sample of size 5 be drawn from the box?

(b) Suppose a box of 100 ipods contains exactly 20 defective ipods. In how many ways can a sample of size 5 with no defective ipods be drawn from this box?

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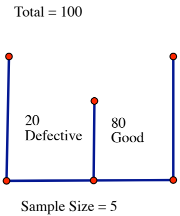

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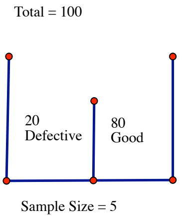

- *The number of samples of different size 5 that we can draw from a box with 100 ipods is*

$$C(100,5) = \frac{100 \cdot 99 \cdot 98 \cdot 97 \cdot 96}{5!}.$$

(b) Suppose a box of 100 ipods contains exactly 20 defective ipods. In how many ways can a sample of size 5 with no defective ipods be drawn from this box?

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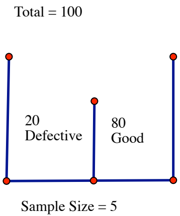

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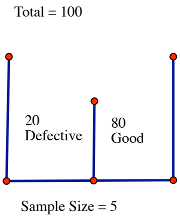

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- *To count the number of possible samples of size 5 from the above box, containing 5 good ipods, we count the number of ways to draw a sample of size 5 from the 80 good ipods in the box. This is* $C(80,5)$.

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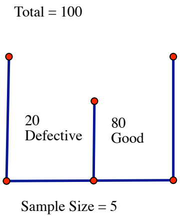

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- *Here we use the complement rule. The number of samples of size 5 with at least one defective equals the total number of samples of size five which are possible minus the number of samples of size 5 which have no defectives(are all good) which equals:* $C(100,5) - C(80,5)$.

13(20 Points) (a) In how many ways can you choose 10 people from the 20 people on your dorm floor to make up a bookstore basketball squad?

(b) For the first game you must assign 7 out of the 10 people on your squad to play on the team and you must assign 2 people to be substitutes. In how many ways can you do this?

(c) In how many ways can you line up your Bookstore basketball squad for a photograph, with the team of 7 standing in the back row and the other 3 squad members sitting in front?

(d) Suppose there are 128 teams in the first round of bookstore basketball. The organizing committee has made a schedule for 64 games with a time and place for each game arranged. In how many ways can the teams be divided into pairs to play the 64 matches. (Each Team plays one match).

13(20 Points) (a) In how many ways can you choose 10 people from the 20 people on your dorm floor to make up a bookstore basketball squad?

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- assign 2 of the remaining teams as substitutes; $C(3,2)$ ways.

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- assign 7 people to the team; $C(10,7)$ ways,
- assign 2 of the remaining teams as substitutes; $C(3,2)$ ways.
- Therefore the entire task can be completed in $\boxed{C(10,7) \cdot C(3,2)}$ ways.

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- Line up the team of 7 in the back row; 7! ways,

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- This task can be broken into two parts
- Line up the team of 7 in the back row; 7! ways,
- Line up the other 3 squad members in front; 3! ways.

(d) Suppose there are 128 teams in the first round of bookstore basketball. The organizing committee has made a schedule for 64 games with a time and place for each game arranged. In how many ways can the teams be divided into pairs to play the 64 matches. (Each Team plays one match).

13(20 Points) (c) In how many ways can you line up your Bookstore basketball squad for a photograph, with the team of 7 standing in the back row and the other 3 squad members sitting in front?

- This task can be broken into two parts
- Line up the team of 7 in the back row; 7! ways,
- Line up the other 3 squad members in front; 3! ways.
- The task can be completed in $\boxed{7! \times 3!}$ ways.

(d) Suppose there are 128 teams in the first round of bookstore basketball. The organizing committee has made a schedule for 64 games with a time and place for each game arranged. In how many ways can the teams be divided into pairs to play the 64 matches. (Each Team plays one match).

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- This is a reverse urn problem with 64 steps. We have 64 organized matches and we need to assign two teams to each.

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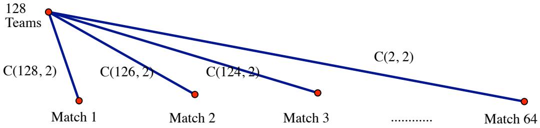

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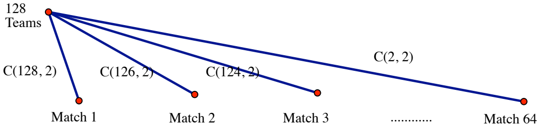

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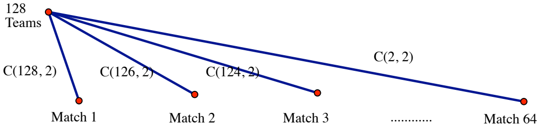

- We can break the task into 64 steps
- Step 1: assign 2 teams to match 1: $C(128, 2) = \frac{128 \cdot 127}{2}$ ways,

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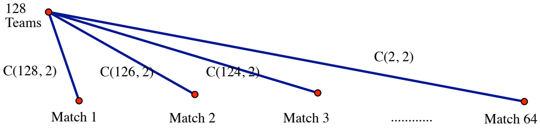

- We can break the task into 64 steps
- Step 1: assign 2 teams to match 1: $C(128, 2) = \frac{128 \cdot 127}{2}$ ways,
- Step 2: assign 2 teams to match 2: $C(126, 2) = \frac{126 \cdot 125}{2}$ ways, ...

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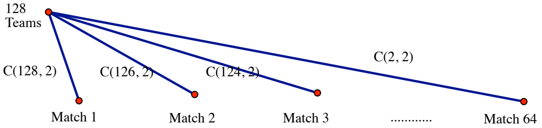

- We can break the task into 64 steps
- Step 1: assign 2 teams to match 1: $C(128, 2) = \frac{128 \cdot 127}{2}$ ways,
- Step 2: assign 2 teams to match 2: $C(126, 2) = \frac{126 \cdot 125}{2}$ ways, ...
- Step 64: assign 2 teams to match 64: $C(2, 2) = \frac{2 \cdot 1}{2}$ ways.

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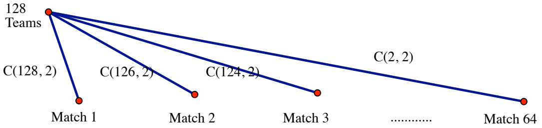

- We can break the task into 64 steps
- Step 1: assign 2 teams to match 1: $C(128, 2) = \frac{128 \cdot 127}{2}$ ways,
- Step 2: assign 2 teams to match 2: $C(126, 2) = \frac{126 \cdot 125}{2}$ ways, ...
- Step 64: assign 2 teams to match 64: $C(2, 2) = \frac{2 \cdot 1}{2}$ ways.
- Therefore the number of ways of completing the task is

$$\boxed{C(128, 2) \cdot C(126, 2) \cdot \cdots \cdot C(2, 2) = \frac{128!}{2^{64}}.}$$

14(10 Pts) (a) If you flip a coin 10 times, how many sequences of heads and tails (of length 10) can result?

(b) How many of the above sequences (resulting from 10 flips of a coin) have exactly 3 Heads?

(c) How many sequences of heads and tails resulting from 10 flips of a coin have exactly 3 tails?

(d) How many sequences of heads and tails resulting from 10 flips of a coin have exactly 8 tails?

(e) How many sequences of heads and tails resulting from 10 flips of a coin have at least 3 tails?

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- $C(10, 8)$

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- $2^{10} - [C(10, 0) + C(10, 1) + C(10, 2)]$